Role of MPC in Building Climate Control

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Abstract
Low energy buildings have attracted a lot of attention in past decades. Recent research is dedicated mainly to optimization of building construction and alternative energy sources. We provide a different approach to the energy-consumption and energy-cost optimization. A generic concept of minimizing energy consumption using current energy sources making use of advanced control techniques is presented. Model Predictive Controller (MPC) presented in this article makes use of both weather forecast and thermal model of a building to control inside temperature. This, by sharp contrast to conventional control strategies such as heating-curve (HC) or rule-based controllers (RBC), enables utilization of thermal capacity of the building. The inside temperature can be maintained at desired levels independent of the outside weather conditions using modified formulation of MPC.

Keywords: model predictive control, costs effectiveness, energy time-management

1 Introduction
Buildings account approximately for 39% of total energy usage Perez-Lombard et al. (2008). Although the energy efficiency of systems and components for heating, ventilating, and air conditioning (HVAC) has improved considerably, there is still potential for substantial improvements. This article provides a comparison of traditional non-predictive techniques and deals with an advanced predictive control technique applied on real building. Widely used control strategy of water heating systems is the weather-compensated control (WCC). This often leads to poor energy management or reduced thermal comfort even if properly set up, because it utilizes current outside temperatures only. Weather conditions, however, can change dramatically in few hours; and due to the heat accumulation in large buildings, it can lead to underheating or overheating of the building easily. The Model Predictive Controller (see Richalet et al. (1976); Kwon et al. (1983); Rawlings and Muske (1993); Zheng and Morari (1994); Campo and Morari (1987)) (MPC) presented in this article introduces a different approach to the heating system control design. As the outside temperature is one of the most influential quantities for the building heating system, weather forecast is employed in the predictive controller. It enables to predict inside temperature trends according to the selected control strategy. The aims of the control can be expressed in natural form as thermal comfort and economic trade off. The paper is organized as follows. Next section briefly discusses currently used strategies, section 3 introduces MPC approach and section 4 provides the case study setup as well as results. The last section concludes the paper.

2 Current Heating Strategies
Let us briefly compare the current major control techniques: on-off room temperature control, weather-compensated control, and PID (Underwood (1999)) with the proposed MPC. The on-off temperature control is the simplest type of control; the heating devices in a room are switched on and off (device state $S$) according to some room temperature error threshold, often with hysteresis as $S = \text{on} \cdot \text{off}(e_t)$. This is a very simple feedback control, which does not contain any information about the dynamics. On the contrary,
the WCC is a feedforward control, which also does not contain any information about the building dynamics. The temperature of the heating medium $t_w$ is set according to the outside temperature $t_{out}$ by means of predetermined HCs $f_{w,c}$. Its advantage is robustness and simple tuning. PID control is one of the most favorite strategies of control engineers (Ang et al. (2005); Li et al. (2006)). The heating water temperature $t_w$ is then determined as $t_w = f_{PID}(e, h)$, where $h$ is a "history". PID controllers are robust and allow accurate tuning, but they cannot reflect the outside temperature effects. This is the reason why PIDs in HVAC control are not as common as in other control applications. Even though all the above controllers are easy to tune for single-input, single-output (SISO) systems, their tuning for multi-variable (MIMO) systems becomes very difficult or even impossible. The PID control can be applied to MIMO systems only in case of specially structured systems. We would therefore appreciate control strategy, which has a feedback, use as much information as possible ($t_{out}$, the weather forecast $t_{pref}$, and others $x$) and include system dynamics as well. These requirements are satisfied by a MPC.

3 Model Predictive Control

MPC is a method for constrained optimal control (Underwood (1999)). In buildings one would aim at optimizing the energy use or costs subject to comfort constraints. Predictions of any disturbances (e.g. internal gains), time-dependencies of the control costs (e.g. dynamic electricity prices), or of the constraints (e.g., thermal comfort range) can be readily included in the optimization. The MPC strategy comprises two basic steps. Firstly, the future control signals are calculated by optimizing the objective function. Secondly, the first component of the control sequence $u(k)$ is sent to the system, whilst the rest is disposed. At the next time instant, new control sequence is calculated (receding horizon). The standard formulation of criterion for MPC can be written as

$$J = \sum_{k=0}^{T-1} q(k) [y(k)^2 - y_r(k)^2] + r(k)u(k)^2,$$

(1)

where $q(k)$ is a weight for difference of the system output $y(k)$ and reference $y_r(k)$, whilst $r(k)$ is a weight of the control signal $u(k)$. By this, the area delimited by the system output below desired value is same as the area above it. This is depicted in Figure 4 by a red line. Such a behavior is not suitable for temperature control of a building. Resulting behavior of the output is delineated by a blue line. This problem can be solved as: a) The intuitive method is to use dynamic weights $q(k)$ and $r(k)$. The complexity of this procedure grows with more reference trajectory levels, but in case of 2 levels, it is the simplest solution. b) In the minimization of the standard criterion, the reference $y_r(k)$ can be substituted with "artificial" reference $w$. This can be done using following convex combination (Camacho and Bordons (1999)):

$$w(t + k) = \alpha w(t + k - 1) + (1 - \alpha)y_r(t + k),$$

(2)

where $w(t) = y(t), k = 1, \ldots, T$ and $\alpha \in (0; 1)$ is a parameter, that determines the smoothness (and speed) of the approaching of the real output to the real reference. c) Reformulation of the part of 1, which refers to the desired value error. If $y(k) < y_r(k)$ then weight the square of this difference using $q(k)$, otherwise the error is not weighted. This problem can be solved using concept of zone control (also called funnel MPČ, Maciejowski (2002)), where the reference error is not weighted in a specified interval while the weighting out is made in a common way.

3.1 Extensions to MPC

In this section we address particular problems in building climate control. As a first example we consider the uncertainty in weather predictions, then we introduce an incorporation of maintenance cost, dynamic electricity prices or contract optimization into the MPC problem. Yet another extension deals with an approximation of MPC for more cost-effective implementation.

- **Stochastic Model Predictive Control.** The main challenge of the overall control problem lies in the uncertainty due to the use of weather predictions. This can be
formulated in two different ways. Either we do not require constraints to be satisfied at all times, but only with a predefined probability (so-called chance constraints), or we explicitly account for the uncertainty in the controller by formulating the future control inputs as functions of future past disturbances Oldewurtel et al. (2008, 2010a).

- **Maintenance cost, dynamic electricity prices and contract optimization.** Besides the energy cost, other criteria could be used when formulating the MPC problem

**Maintenance costs.** When the lifetime of the appliances is considered, the maintenance cost $l^m_k(x_k,u_k)$ can be added into the cost function. Three general cases can be distinguished: a) Linear dependence on usage, where cost is the sum of energy cost and maintenance cost multiplied by the usage of the appliance, i.e. $l^m_k(x_k,u_k) = c_l u^{ml}_k$, where $c_l$ is maintenance cost and $u^{ml}_k$ is linearly dependent on usage. b) Life-time dependence which leads to a Mixed-Integer-Problem. Let us denote $u^{mo}_k$ an integer input variable expressing if a plant is on $u^{mo}_k = 1$ or off $u^{mo}_k = 0$. Life-time dependent cost can be expressed as $l^m_k(x_k,u_k) = c_o u^{mo}_k$, where is $c_o$ is life time cost. c) Switching dependence, where the maintenance cost depends on the number of on-off switches, i.e. $l^m_k(x_k,u_k) = c_s |u^{mo}_k - u^{mo}_{k-1}|$, where $c_s$ is a switching cost.

**Dynamic electricity prices and reduction of peak electricity demand**
The minimization of total electricity consumption seems can lead to the undesirable peaks in electricity demand. The building envelope constitutes a thermal storage which poses possibility to shift electricity demand from high price to low price times or from high loading to low loading times, respectively. This is readily possible, when the electricity prices of the day-ahead spot market price are used in the MPC problem Oldewurtel et al. (2010b).

- **Approximations and explicit MPC.** The real implementation involves having appropriate hard-and software at the building site for solving the optimization problems at each time step. In order to keep investment costs low, one could think of a prior store of the control law, i.e. in real operation the controller only needs to evaluate the current state and weather and internal gain prediction and from this decide which control input to apply. This control law could be determined by using a model of the real building and controlling it in simulation with an MPC controller and storing the computed control inputs for different weather and occupancy regimes. In a second step, learning techniques can be used in order to distinguish different regimes of operation that result in different control laws. Depending on how many regimes you allow, the solution will be an approximation to the original online-optimization and thus degrade in performance.

### 4 Case Study

The presented MPC scheme of Problem 1 was applied to the building of the Czech Technical University (CTU) in Prague (see Figure 4), which is composed of several blocks with the same construction and way of use. The heating system scheme of one building block is depicted in Figure 4. Detailed description of the heating system and modeling can be found in Široký et al. (2010); Cigler and Prívara (2010).

**Description of the controller.** There are several requirements to be fulfilled: Reference tracking. The reference trajectory $y_{r,k}$ (room temperature) is known as a schedule. The major advantage of MPC is the ability of computing the outputs and corresponding input signals in advance, that is, it is possible to avoid sudden changes in the control signal and undesired effects of delays in the system response. The schedule defines two levels of the room temperature: 22°C during the day and 19°C at night and over the weekends. The reference tracking should be perfect for the upper level from its beginning to ending edge, while the lower reference level should represent a temperature below which the temperature should not fall. Thus, we proposed an alternative MPC problem formulation - the displacement is penalized only below the reference trajectory, see Figure 4. 2-norm for
accurate performance is used. *Minimization of energy consumption.* As the return water \( \vartheta_{rw} \) circulates in the heating system (Figure 4), the energy consumed by the heating-up of the building is linearly dependent on the positive difference between \( \vartheta_{sw} \) and \( \vartheta_{rw} \) entering/exiting the three port valve. Thus, the 1-norm of weighted inputs is to be minimized.

**MPC problem formulation.** The standard state space system is partitioned as follows:

\[
x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k + Du_k, \quad z_k = Vx_k + Wu_k,
\]

where \( y_k \) stands for outputs with reference signal (e.g. \( \vartheta_{in,k} \)), \( z_k \) are input-output differences, i.e. \( z_k = \vartheta_{sw,k} - \vartheta_{rw,k} \). The weighting of the particular variables is carried out by adding the slack variables \( a_k \) and \( b_k \). The resulting optimization problem can be written as:

\[
J = \min_{a_k, b_k, u_k} \sum_{k=0}^{N-1} a_k^T Q a_k + R b_k,
\]

\[
y_k = CA^{k-1} x_0 + \sum_{i=0}^{k-1} CA^{k-1-i} Bu_i + Du_k, \quad z_k = VA^{k-1} x_0 + \sum_{i=0}^{k-1} VA^{k-1-i} Bu_i + Wu_k,
\]

\[
y_{r,k} - y_k - a_k \leq 0, \quad a_k \geq 0, \quad z_k - b_k \leq 0, \quad b_k \geq 0, \quad u_{\min} \leq u_k \leq u_{\max}, \quad |u_k - u_{k-1}| \leq \Delta u_{\max}.
\]

**Setup and results.** Several comparisons of the real building experiments have been performed. The first comparison (cross comparison) using \( B_1 \) and \( B_2 \) blocks had two phases, each lasted for a week. In the first week, \( B_1 \) was controlled by the HC and block \( B_2 \) by MPC. The other week, the control strategies were switched. The second comparison uses heating degree days (HDD) for the normalization of the building energy consumption which is defined as \( \text{HDD} = \sum_{k=\text{begin}}^{\text{end}} y_{r,k} - \vartheta_{o,k} \), where \( \text{begin} \), \( \text{end} \) denote the beginning and the end of the measured period, respectively. To minimize the negative effect of different weather conditions, time periods with similar average temperature were used. Due to constant heating water flow, the energy consumption measure (denoted as \( E_{\text{CM}} \)) is:

\[
E_{\text{CM}} = \sum_{k=\text{begin}}^{\text{end}} (\vartheta_{sw,k} - \vartheta_{rsw,k}) + (\vartheta_{swn,k} - \vartheta_{rwn,k})
\]

The Crittall heating system utilizes the building mass as a thermal storage, which enabled MPC to preheat the concrete mainly at night. The beneficial side effect of MPC strategy
Table 1. Comparison of HC and MPC strategies using similar building blocks $B_1$ and $B_2$.

<table>
<thead>
<tr>
<th></th>
<th>$B_1$ mean $\vartheta_o^{\circ}C$</th>
<th>$B_1$ mean $\vartheta_s, \vartheta_n^{\circ}C$</th>
<th>$B_2$ mean $\vartheta_s, \vartheta_n^{\circ}C$</th>
<th>MPC savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st week</td>
<td>$-3.4$</td>
<td>$21.4$</td>
<td>$21.1$</td>
<td>$15.54%$</td>
</tr>
<tr>
<td>2nd week</td>
<td>$-1.3$</td>
<td>$21.4$</td>
<td>$20.9$</td>
<td>$16.94%$</td>
</tr>
</tbody>
</table>

was a significant energy peak reduction as can be seen in Figure 4. The cross comparison results are summarized in Table 1 with MPC savings approximately 16% of energy in both weeks. The results from HDD based comparison are in Table 2. The relative savings were more significant at insulated building blocks $B_1$ and $B_2$.

Table 2. Heating degree days based comparison. The ratio $E_{CM}/HDD$ expresses normalized energy demands for heating.

<table>
<thead>
<tr>
<th></th>
<th>$E_{CM}/HDD$</th>
<th>$\text{mean } \vartheta_o^{\circ}C$</th>
<th>$\text{mean } \vartheta_s, \vartheta_n^{\circ}C$</th>
<th>days compared</th>
<th>relative MPC savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$ - HC</td>
<td>0.906</td>
<td>3.8</td>
<td>21.6</td>
<td>84</td>
<td>28.74 %</td>
</tr>
<tr>
<td>$B_1$ - MPC</td>
<td>0.645</td>
<td>3.2</td>
<td>21.8</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>$B_2$ - HC</td>
<td>0.813</td>
<td>4.0</td>
<td>21.7</td>
<td>85</td>
<td>26.83 %</td>
</tr>
<tr>
<td>$B_2$ - MPC</td>
<td>0.595</td>
<td>3.0</td>
<td>21.7</td>
<td>49</td>
<td></td>
</tr>
</tbody>
</table>

5 Conclusion and Acknowledgements

MPC was operational Jan-March 2010 and has proven significant savings. This work has been supported in the scope of grant No. FR-TI1/517, "Control systems for energy consumption optimization in low-energy and passive houses".

References